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ON THE FUTURE CONTRACT QUALITY OPTION: A NEW LOOK [1]

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Abstract

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Keywords: *Future Quality Option, Asset Pricing, German Bund.*

JEL Classification: G13.

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On the future contract quality option: A new look

by

Alejandro Balbás [1] and Susana Reichardt [2], [3]

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I. Introduction

This paper deals with the quality option usually embedded in future contracts. According to Cox *et al.* (1981), future contracts may incorporate four kinds of embedded options: The quality option (the future seller chooses the security to deliver amongst a set of deliverable assets), the quantity option (the future seller chooses the quantity of the underlying asset to deliver), the temporary option (the future seller chooses the date within a time interval) and the localization option (the seller chooses the place). Some future contracts simultaneously incorporate several options.

We will focus on the quality option of future contracts on bonds, although the developed methodology also applies for more complex securities. Bond futures have a notional underlying asset and, consequently, the market organizers have to provide a list of deliverable bonds. A new flotation before the future expiration may provoke the enlargement of the list, and the future seller will decide at the future maturity the bond that he/she prefers to deliver.

The future buyer has no choice with respect the asset he/she will receive, and therefore he/she merits compensation. Hence the price of the future contract decreases and the detected fall has been the key used by many authors to price the embedded quality option. This price will critically depend on the volatility of the deliverable securities. If, as usual, they are bonds, then their volatility will be small and so will the quality option price. But small price does not imply negligible price. On the contrary, as stated in Chance and Hemler (1993), to ignore the quality option may lead to important errors when composing hedging strategies and may underestimate many risk premiums or several measures of market efficiency see (Kamara 1990, Ahn *et al*. 2002, Buhler and Düllmann 2003 or Merrick *et al.* 2005).

Following the analysis of Chance and Hemler (1993), there are several alternatives to price the quality option. So, Margrave (1978), Gay and Manaster (1984) Boyle (1989), Hemler (1990), Bellier (1997) or Anderson and Martínez-Garmendia (1999), develop a theory to price the option allowing its buyer to change two previously fixed securities. Another possibility consists in pricing the option as the profit obtained by the future seller due to the difference of prices between the

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bond he/she finally delivers and the one he/she would deliver when the future was sold (Kane and Marcus 1986, Barnhill 1990, Hedge 1990, Hemler 1990 or Stickland 1992). The most usual way prices the option at any date before the future maturity as the difference between the theoretical future price of the cheapest to deliver bond and the future price reflected by the market (Hedge 1990, Hemler 1990, Stickland 1992, Yu 1997). We will also follow this approach though, as will be justified, we will not draw on the cheapest to deliver asset. The last method indicated by Chance and Hemler prices the option by using the cash flows of a roll-over strategy that buys a (theoretical) future contract on the cheapest to deliver bond and sells the future contract (Barnhill and Seale 1988, Barnhill 1990, Hedge 1990 or Yu 1997 and 1999).

Recent papers (Chen 1997, Carr and Chen 1997, Bick 1997) draw on the third method above and classic dynamic models of the *TSIR* behaviour (Vasicek 1977, Cox *et al.* 1985) to price quality options. More complex models of the *TSIR* dynamics (Hull and White 1990, Heath *et al.* 1992, etc.) are used in Lin and Paxon (1993) and (1995), Ritchken and Sankarasubramanian (1992) and (1995), Yu (1997), Chen *et al*. (1999) or Nunes and Ferreira (2003).

This paper will attempt to price the quality option by drawing on the standard methods of Pricing Theory, but the imposed assumptions will be as simple as possible. Firstly, a precise definition of the quality option will be yielded, and it will be justified that the number of embedded quality options in a future contract equals the number of deliverable assets. Secondly, it will be proved that the quality options may be replicated with the available securities by means of a static portfolio, *i.e.*, the replicating portfolio does not have to be rebalanced till the future maturity. Thirdly, a simple arbitrage-linked argument allows us to provide the quality option with the price of its replica.

To deal with a static replica seems to reveal several advantages. Indeed, our results are robust with regard to any dynamic assumption concerning the *TSIR* behaviour. Moreover, the static replica permits us to introduce transaction costs in a simple manner, so that they can be considered to price the replicating portfolio and the quality option. Finally, the replicating portfolio is not unique, since the future contract is close to the difference between its calls and puts. Clearly, the future contract options are affected by the presence of the quality option, so they contain information that may be quite interesting when pricing the quality option. Using future contract options makes the analysis and the empirical results much more robust because they have to overcome different tests based on different replicating strategies.

It is also worth to mention a final difference with respect to previous literature. As already said, there are more than one quality options per future contract. Authors usually price the cheapest one in order to point out that the quality option effect is not so high.[4] However we have priced the most expensive quality option. It may be justified because, as said above, the quality option may be replicated in a static framework and, consequently, it is available to traders. We have considered that an almost never studied available security may deserve our attention. Anyway, it is convenient to indicate that our new methodology similarly applies to price the cheapest embedded quality option.

Papers outline is as follows. The second section will present the theoretical results and the methodology. We will price the option in both perfect and imperfect markets.[5] We will yield several closed formulas related to the securities we are using when replicating the quality option (the future contract or its calls and puts). The third section deals with an empirical test implemented with the *German Bund* traded in *EUREX*. There are two analyzed periods. The first one focuses on the future contract with maturity in December 2002, and the quality option was priced between September the second and December the sixth 2002 (last trading date). The second analysis considers the future contract with maturity in December 2005, and the quality option was

[4] Notice that if the quality option value were really negligible then it would be complex to understand some speculative behaviours pointed out by several authors (*e.g*., Merrick *et al.* 2005).
[5] As usual, in a market with frictions, the exact price must be replaced by upper and lower bonds of the theoretical price.

priced from November the second to November the eighteenth. Our first test did not use the future calls and puts and three months before maturity the quality option approximate average value equalled 2% of the future contract nominal value. This is much larger than usual (recall that we are pricing the most expensive embedded option).[6] The quality option price decreased with time. This effect may affect the derivatives of the future contract and, therefore, as said above, to ignore the quality option presence may cause other pricing errors (Ronn and Bliss 1994 or Cherubini and Exposito 1995, among others, have proposed a pricing method for options on futures with embedded options). With regard to the second tested period, we have used the future contract calls and puts. The quality option price average value was 2.5% three months before maturity and was decreasing slowly. For the first period, the quality option price was around 1% one month before maturity. There are two factors that could explain the difference: 1) Deliverable bonds of the future contract with maturity in December 2002 are the same coupon, whereas deliverable bonds of the future with maturity in December 2005 are different coupons. 2) After the second tested period a new bond was added to the list of deliverable bonds.

The last section of the paper presents the major conclusions, and several tables and figures illustrate the results of the empirical test.

II. Replicating and Pricing the Quality Option

The quality option will be replicated and priced by drawing on the classical static approach of Financial Economics. Firstly we will not incorporate frictions. Thus, consider the current date t_0 =0 and a future one denoted by T. There are n risky securities $S_1, S_2, ..., S_n$, a riskless asset, whose interest rate between 0 and T is represented by r, and a future contract F with maturity at T and whose underlying assets are $S_1, S_2, ..., S_n$, where the quality option will be exercised by the future seller. The (numerical) initial price of S_j is denoted by $p_{0,j}$, $j = 1,2,...,n$, and f is the initial future price. The (random) final price of S_j will be p_j, $j = 1,2,...,n$, and f^* will denote the future price at maturity.

There is a conversion factor $\delta_j > 0$ that affects S_j, $j = 1,2,...,n$, so the pay-off received by the future seller at maturity is given by

$$f - f^* + \delta_i f^* - p_i$$

if he/she delivers S_i. Of course, to prevent the existence of arbitrage at T the expression

$$0 = (\delta_i f^* - p_i) \geq (\delta_j f^* - p_j) \quad \forall j$$

must hold, and we get

$$f^* = \frac{p_i}{\delta_i} \leq \frac{p_j}{\delta_j}, \quad \forall j \ . \qquad (1)$$

Therefore, the final pay-off of the future seller becomes $f - f^*$. Security S_i is usually called the cheapest asset to deliver.

Next let us construct a new strategy replicating the sale of the previous future contract. So, fix *i* amongst $j = 1,2,...,n$, and consider the derivative contract F_i allowing the seller to deliver

[6] For instance, Chen *et al.* (1999) estimated a quality option value around 0.02% and Nunes and Ferreira (2003) estimated value was around 0.05%.

$1/\delta_i$ units of S_i at T for f monetary units. Let Q_i be the option permitting the buyer to receive $1/\delta_i$ units of S_i at T if he/she delivers $1/\delta_j$ units of the chosen security S_j, that belongs to the set $\{S_1, S_2, ..., S_n\}$.

Proposition 1. *The sale of F may be replicated by sale of F_i and the purchase of Q_i, i=1,2,…,n.*

Proof. The sale of F_i will pay $f - \frac{p_i}{\delta_i}$ and the purchase of Q_i will pay $\frac{p_i}{\delta_i} - \underset{j}{Min}\frac{p_j}{\delta_j}$ at T. Bearing in mind (1), the combination of both strategies will pay $f - f^*$, pay-off associated with a short position in F. ■

Remark 2. According to the statement above the sale of F incorporates *n* implied quality options $Q_1, Q_2, ..., Q_n$. Each option Qj is associated with security S_j, $j = 1,2,...,n$. ■

Proposition 3. *The price of Q_j is given by*

$$q_j = \frac{p_{0,j}}{\delta_j} - \frac{f}{(1+r)^T} \qquad j = 1,2,...,n \tag{2}$$

Proof. The Law of One Price (LOP) and Proposition 1 lead to $0 = A_j + q_j$ where A_j is the price of a short-sale of F_j. Thus, it is sufficient to show that $A_j = \frac{f}{(1+r)^T} - \frac{p_{0,j}}{\delta_j}$ which is obvious since the sale of F_j is replicated by lending $\frac{f}{(1+r)^T}$ monetary units and selling $1/\delta_j$ units of S_j. ■

Remark 4. Expression (2) clearly points out that all the implied quality options do not necessarily have the same price. We will consider the most expensive one in order to introduce "the quality option price", *i.e.*,

$$q = \underset{j}{Max}\left(\frac{p_{0,j}}{\delta_j} - \frac{f}{(1+r)^T}\right) \tag{3}$$

will be the value that we will estimate in our empirical test.

The literature has focused on the quality option associated with the option that the future seller would deliver if the decision were made at the initial date t_0=0, and it may be easily proved that this option price is given by (3) if the maximum is replaced by the minimum value. Thus, Definition (3) is a important difference with respect to previous works. However, we prefer to concentrate on the option with the highest value because this is also implied by the future contract F, in the sense that it can be also replicated by using F (Proposition 1), and the empirical analysis will reflect that its value is not negligible. After words, all the implied options of Remark 2 may merit our attention and a major objective of this paper is to show that the existence of several underlying assets for interest rate future contracts may lead to the existence of significant quality options.

Despite the comments above it is worth to indicate that our methodology and its implications also apply if "*Min*" substitutes "*Max*" in (2).

Finally, notice that the list of deliverable assets is often open in practice, in the sense that before T the market organizers can add new securities to the set $S_1, S_2, ..., S_n$. Furthermore, this is the case when dealing with the *Bund Future Contract*, the one we will empirically check. Nevertheless, if we assume that the list of deliverable assets may be enlarged in the same manner when analyzing the position of the Q_i option buyer, then Proposition 1 still holds, and the proof is absolutely similar and therefore omitted. Then we have: ■

Proposition 5. *Proposition 1 still holds if the set of deliverable securities may be enlarged before* T . ■

Let us now assume that there exist transaction costs given by the usual bid/ask spread. Suppose that f_a, f_b $(f_a \geq f_b)$ and $p^a_{0,j}, p^b_{0,j}$ $(p^a_{0,j} \geq p^b_{0,j})$, $j = 1,2,...,n$, are the ask and bid prices at *t=0*. Let r_a and r_b $(r_a \geq r_b)$ be the borrowing and lending interest rate between. t_0 and T . We will not consider frictions at the second date.

Proposition 6. *The upper and lower bounds below must hold*

$$\frac{p^b_{0,j}}{\delta_j} - \frac{f_a}{(1+r_b)^T} \leq q_j \leq \frac{p^a_{0,j}}{\delta_j} - \frac{f_b}{(1+r_a)^T} \qquad j = 1,2,...,n \quad (4)$$

Proof. First of all,

$$q_j - \left(\frac{p^b_{0,j}}{\delta_j} - \frac{f_a}{(1+r_b)^T} \right) \geq 0, \; j = 1,2,...,n,$$

i.e., if one buys the quality option and sells its replica there are no positive incomes. Besides,

$$\frac{p^a_{0j}}{\delta_j} - \frac{f_b}{(1+r_a)^T} - q_j \geq 0, \; j = 1,2,...,n,$$

i.e., if one buys the quality option replica and sells the option we cannot expect any positive income. ■

Remark 7. Firstly, notice that Proposition 6 extends Proposition 3. Secondly, both expressions must be slightly modified if S_j pays the dividend (or coupon) d_j at τ_j $(t_0 \leq \tau_j \leq T)$. If so,

$$\frac{p^b_{0,j}}{\delta_j} - \frac{f_a}{(1+r_b)^T} - \frac{d_j}{\delta_j (1+r^b_{\tau_j})^{\tau_j}} \leq q_j \leq \frac{p^a_{0,j}}{\delta_j} - \frac{f_b}{(1+r_a)^T} - \frac{d_j}{\delta_j (1+r^a_{\tau_j})^{\tau_j}} \qquad (5)$$

for $j = 1,2,...,n$, and the proof is absolutely similar and therefore omitted. Thirdly, all the expressions hold if more deliverable assets may be added before T . ■

Next we will develop the methodology allowing us to draw on the information contained in calls and puts on the future contract in order to price the quality option. Therefore, along with the securities above, we will consider the existence of American calls and puts with the same strike *X* and maturity at T' $(T' < T)$. Since the empirical test will draw on the quality option implied in the *German Bund*, we will consider the properties of its future options. Hence, the calls and puts above are "pure options", *i.e.*, the premium will be paid at T' or when the option is exercised if there is early exercise.[7]

Proposition 8. *Suppose that there are no frictions and denote by* c *and* p *the call and put price, respectively. Then,*

$$q_j = \frac{p_{0,j}}{\delta_j} + \frac{p - c - X}{(1+r)^T} \qquad k = 1,2,...,n \tag{6}$$

Proof. The put-call parity relationship for European or American "pure options" (see Lieu, 1990) leads to

$$p - c = X - f$$

Thus, Expression (6) trivially follows from (2). ■

Remark 9. As in Remark 7, if S_j pays the dividend (or coupon) d_j at τ_j $(t_0 \le \tau_j \le T)$ then the quality option price satisfies

$$q_j = \frac{p_{0,j}}{\delta_j} + \frac{p - c - X}{(1+r)^T} - \frac{d_j}{\delta_j (1 + r_{\tau_j})^{\tau_j}} \qquad j = 1,2,...,n \tag{7}$$

r_{τ_j} being the risk-free rate between t_0 and τ_j. The expression also holds if the set of deliverable securities may grow before T . ■

Remark 10. If the future options were European and the option premium were paid at t_0 then, with the obvious notations, we would have

$$q_j = \frac{p_{0,j}}{\delta_j} + \frac{(p_c - c_c)(1 + r')^{T'} - X}{(1+r)^T} \qquad j = 1,2,...,n \tag{8}$$

Indeed, in such a case the put-call parity becomes (see Lieu, 1990)

$$p_c - c_c = \frac{X}{(1+r')^{T'}} - \frac{f}{(1+r')^{T'}} ,$$

and (8) trivially follows. ■

[7] See Duffie (1989) or Lieu (1990) for further details about this kind of option.

Next let us assume the existence of transaction costs. Let c_a, c_b $(c_a \geq c_b)$ and p_a, p_b $(p_a \geq p_b)$ be the ask and the bid prices of the call and the put option. Then, one has:

Proposition 11. *The inequalities below must hold*

$$\frac{p_{0j}^{b}}{\delta_j} + \frac{p_b - c_a - X}{(1+r_a)^T} \leq q_j \leq \frac{p_{0j}^{a}}{\delta_j} + \frac{p_a - c_b - X}{(1+r_b)^T} \qquad j = 1,2,\ldots,n \tag{9}$$

Proof. According to Jouini and Kallal (1995), the absence of arbitrage in a market with frictions implies the existence of an (ideal) arbitrage-free frictionless market whose prices lie within the bid/ask spread. Thus, there exist $p_{0,j}$, $j = 1,2,\ldots,n$, c, p, and a risk-free rate r such that $p_{0j}^{b} \leq p_{0j} \leq p_{0j}^{a}$, $j = 1,2,\ldots,n$, $c_b \leq c \leq c_a$, $p_b \leq p \leq p_a$, $r_b \leq r \leq r_a$ and (6) holds. Whence, (9) becomes obvious.[8] ■

Remark 12. Once again, the presence of dividends d_j at τ_j $(t_0 \leq \tau_j \leq T)$ leads to:

$$\frac{p_{0j}^{b}}{\delta_j} + \frac{p_b - c_a - X}{(1+r_a)^T} - \frac{d_j}{\delta_j (1+r_{\tau_j}^{b})^{\tau_j}} \leq q_j \leq \frac{p_{0j}^{a}}{\delta_j} + \frac{p_a - c_b - X}{(1+r_b)^T} - \frac{d_j}{\delta_j (1+r_{\tau_j}^{a})^{\tau_j}} \tag{10}$$

$j = 1,2,\ldots,n$. Moreover, an additional flotation would not modify the formulas. ■

III. Empirical Test: Data and Results

We used the *German Bund Future Contract*, available in *EUREX*, to test the quality option price. The underlying asset is a notional bond issued by the German government whose annual coupon equals 6%. The contract nominal value is 100000 euros and prices represent a percentage of the nominal value with two decimal digits.[9]

There are four available maturities, March, June, September and December, although the shortest one reflects the far highest activity. The future contract can be traded until one day before its maturity, at 12:30 p.m. The delivery must take place on the tenth day of the delivery month, and the deliverable assets are bonds issued by the German government with maturity between 8 and 11 years. The set of deliverable assets may enlarge, if a new flotation occurs and the new bonds satisfy some required conditions.

We have addressed two empirical tests. Both analyses draw on high frequency perfectly synchronized data in order to price the quality option with the highest possible precision.[10] The first one does not use future options and focuses on the future contract with maturity in December

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[8] Bearing in mind (8), similar arguments allow us to obtain upper and lower bounds for the quality option price if the future options are not "pure options"

[9] Table 1 presents a synopsis of the results of previous empirical studies. It has been provided for two reasons: Firstly, it allows us to compare different results. Secondly, it may be seen that the *German Bund* is not the most usual focus of empirical papers. However, the *German Bund Future* presents an interesting property since it only contains quality options, and no more options are simultaneously embedded.

[10] We follow the ideas and precision of the empirical study of Balbás *et al.* (2000), where the level of integration between the Spanish spot and derivative markets is verified by using a similar database.

2002. The quality option price has been computed between September the second and December the sixth (last trading day). In order to use perfectly synchronized data we only priced the option at those minutes such that we had all of the involved prices.[11] Minute by minute we have priced the quality option by considering bid and ask prices of the set of deliverable bonds and the future contract. We have also distinguished between borrowing and lending rates and the whole set of data has been provided by Bloomberg.

The future contract presents three quality options.[12] The conversion factor is the bond price per unit of nominal value, at the future expiration and under a flat TSIR equal to the notional bond coupon (6% if we deal with the *German Bund*).

First of all we computed the price of the three quality options under the frictionless assumption. At every minute we took average values of the bid and ask prices for all the involved securities, including the risk-free rate. Minute by minute the highest price corresponded to the quality option associated with the bond with longest maturity. According to (3), this is the quality option price that we measured.

Table 3 provides daily average values of the quality option, which clearly decreases and shows a negative slope (thirteen weeks before maturity the quality option value equals 2% of the future nominal value, whereas one week before expiration it falls to 0,6%).

In a second step, we incorporated transaction costs and estimated the upper and lower bounds of the quality option price. We always obtained that both bounds were associated with the bond with longest maturity. Furthermore, the three spreads showed void intersection.

Tables 4, 5, 6 and 7 yield daily average values for the bounds. Figures 1, 2, 3, 4, 5 and 6 provide the dynamic evolution of both the quality option price in a frictionless world and the bounds in a world with frictions. It is easy to check the stability of the distance between the quality option price and its bounds. The difference between the upper bound and the price almost equal the difference between the price and the lower bound (they usually lie within the spread 35 - 55 euros). Figures 4, 5 and 6 extend the information.

Our second analysis involves pure options on the future contract. The options can be traded at any date before its expiration. Our study deals with pure options whose underlying future matured in December 2005. Table 8 summarizes the deliverable bonds properties. The quality option price was obtained from November the second to November the eighteenth, 2005. We took the strikes 119, 119.5, 120 and 120.5, since our database contained its premiums perfectly synchronized with the remainder variables.[13] The whole database was provided by Bloomberg

Firstly we computed the three quality option values in a frictionless world. The result is similar to that obtained when dealing with the future contracts rather than their pure options, in the sense that the quality option value rises if so does the associated bond maturity. We follow (3) to define the (global) quality option value, and Tables 9, 10, 11 and 12 yield average values of the quality option price, which is usually close to 2.5% of the nominal.

Then we considered transaction costs and computed bounds of the quality option price. Once again the bounds are given by the bond with highest maturity, and the deliverable bonds provided spreads with empty intersection. Tables 13, 14, 15 and 16 present daily average values of the lower bound, while Tables 17, 18, 19 and 20 give upper bounds.

Figures 7, 8 9 and 10 show the dynamic evolution (fall) of the quality option price and its bounds. The distance between the price and its bounds is stable and lies within the spread 50 – 60 euros.

[11] We had the bonds prices and the future price minute by minute, but we did not get the interest rates. Thus, several minutes have been removed and our analysis involved 1250 minutes.
[12] There were three involved bonds. There was not any new flotation before the future expiration. See Table 2.
[13] We used the strike 119 to price the quality option in 86 minutes,119,5 was used in 182 minutes, 120 in 161 minutes and 120,5 in 92 minutes. The remainder strikes were not used due to the scarce number of minutes that we could have studied.

Overall, the results are coherent and robust, in the sense that the existence of four different strikes does not generate contradictions. On the contrary, every strike yields additional information with respect to the remainder ones.

IV. Conclusions

The quality option embedded in many future contracts may be replicated by using a static approach. It allows us to provide several replicating portfolios since future calls and puts may be incorporated. Furthermore, the static analysis makes it far easier to bear in mind transaction costs when pricing the quality option.

The results of every empirical analysis based on the static approach seem to be very robust. Indeed, they do not depend on any dynamic hypothesis, they have to overcome several test due to the existence of different replicating portfolios, they can be obtained from perfectly synchronized real market data and they can incorporate imperfections and the information contained in a large set of assets.

Despite the methodology applies for future contracts on quite different sort of securities, we have empirically tested a bond market, since this is the most usual case in practice. We have checked the quality option of the *German Bund Future Contract*. Three months before maturity the (most expensive) embedded quality option approximate average value lies within the spread [1.9%, 2.8%], which is far of being a negligible price. This may justify that, as pointed out by other authors, the presence of quality options has to be considered when pricing future derivatives and testing the market efficiency. To ignore this presence may provoke speculative strategies trying to benefit from possible market inefficiencies.

TABLE 1

Empirical papers pricing the quality option (we have not included those studies providing the global price of several embedded options. For instance, Labarge 1988, Hedge 1988 and 1989 or Gay and Manaster 1991)

Paper	**Future Contract**	**Period or Maturity**	**Methodology and average option price** (as a percentage of the nominal value)
Kane and Marcus (1986)	(*U.S. Treasury Bond*)	Sept. 1981, March 1982, Sept. 1982, March 1983	2.365 (M4)
Barnhill and Seale (1988)	(*U.S. Treasury Bond*)	Dec. 1977 – Dec.1984	1.1918; 0.28112 (M5)
Barnhill (1990)	(*U.S. Treasury Bond*)	Dec.1977 – Dec.1984	0.25; 0.168; 0.117; 0.085 (M4) 1.191; 0.632; 0.281; 0.135 (M5)
	(*U.S. Treasury		0.464 (M3)

Hedge (1990)	*Bond*)	Dec. 1977 – Dec. 1986	0.329 (M4) 0.209 (M5)
Hemler (1990)	(*U.S. Treasury Bond*)	1977 – 1986	0.713 (M1 2 assets) 1.243 (M1 3 assets) 0.126 (M3) 0.245 (M4)
Stickland (1992)	(*Long Gilt*)	March 1987– Dec. 1988	0.214 (M3) 0.227 (M4)
Lin and Paxson (1993)	(*German Government Bond*)	March 1989 – Dec. 1991	0.095 (M2)
Ritchken and Sankarasubramanian (1995)	(*U.S. Treasury Bond*)	Sept. 1990	2.5642 (M2)
Yu (1997)	(*Japan Government Bond*)	Dec.1989 – March 1994	0.121 (M2) 0.161 (M3) 0.083 (M5)
Chen, Chou and Lin (1999)	(*Japan Government Bond*)	June 1990 – March 1994	0.021 (M2)

M1: Methodology based on Margrave (1978).
M2: Methodology based on a dynamic model for the *TSIR* behaviour.
M3: The quality option value is the difference between a future contract on the cheapest bond to deliver and the product of that bond conversion factor and the future price.
M4: The quality option price is the difference between the future seller earnings if he/she delivers the cheapest bond at maturity instead of the cheapest bond at the future contract sale.
M5: The quality option value is given by the earnings of a roll-over strategy holding at any instant the cheapest bond.

TABLE 2
Deliverable bonds (first study)
(future maturity in December, 2002)

Coupon	Coupon payment	Maturity	Conversion factor
5%	July, 4th	July, 4th, 2011	0.934161
5%	January, 4th	January, 4th, 2012	0.931496
5%	July, 4th	July, 4th, 2012	0.928434

TABLE 3
Quality option price (euros) (first study)
(frictionless case)

	Average	Standard D.	Min	Max
10/9/02	2067.9334	13.9782	2047.7861	2088.7609
17/9/02	2044.1395	17.8446	2017.4190	2092.8465
24/9/02	1982.6329	20.4426	1949.4674	2007.8865
1/10/02	1811.3299	23.8257	1750.9252	1847.4629
8/10/02	1585.5814	10.3722	1565.1919	1605.9701
15/10/02	1356.7231	15.3214	1332.9479	1386.5163
22/10/02	1193.6023	8.8620	1179.2355	1214.0692
29/10/02	1070.6210	11.9366	1049.2000	1089.9969
5/11/02	919.9963	11.2303	898.5432	945.9233
12/11/02	904.6217	7.0179	893.6162	916.8602
19/11/02	802.0046	8.0243	791.8705	816.0660
26/11/02	663.4569	10.1989	644.7042	682.3199
3/12/02	602.2256	9.1027	575.1777	619.7900

TABLE 4
Lower bounds for the quality option price (euros) (first study)
(first bid/ask within the minute)

	Average	S.D.	Min.	Max.
10/09/2002	2018.4435	19.5208	1978.7430	2047.7374
17/09/2002	1997.7596	17.7651	1977.7140	2047.8474
24/09/2002	1934.7991	23.4806	1898.1506	1973.6457
01/10/2002	1764.9097	30.3892	1706.8899	1809.8156
08/10/2002	1541.2431	16.2681	1513.3110	1575.7608
15/10/2002	1310.2976	16.1995	1280.3448	1349.6178
22/10/2002	1150.5683	12.0879	1124.7260	1178.1156
29/10/2002	1030.2980	18.3060	995.8008	1058.8919
05/11/2002	879.8488	14.7958	850.1267	905.9526
12/11/2002	860.0444	14.8944	823.5620	879.8748
19/11/2002	767.4107	9.7719	749.8169	782.1294
26/11/2002	625.5274	14.4004	596.3347	654.7039
03/12/2002	564.0362	14.7344	527.3347	595.2600

TABLE 5
Lower bounds for the quality option (euros) (first study)
(last bid/ask within the minute)

	Average	S.D.	Min.	Max.
10/09/2002	2017.7458	14.3013	1994.9835	2037.8175
17/09/2002	1998.1446	15.1185	1969.6991	2039.6163
24/09/2002	1934.9169	21.7468	1905.2398	1970.4589
01/10/2002	1765.9666	27.1355	1685.5423	1799.8779
08/10/2002	1540.7792	14.0460	1497.1262	1562.9980
15/10/2002	1312.1453	16.5660	1281.1645	1342.1267
22/10/2002	1150.6190	11.1744	1122.1666	1169.4437
29/10/2002	1027.0920	12.4917	999.6602	1040.2305
05/11/2002	876.0686	11.8677	851.2801	895.9836
12/11/2002	866.8531	8.9738	851.1142	887.7646
19/11/2002	762.2885	5.7949	753.1992	772.1482
26/11/2002	626.4354	8.0993	616.3092	643.1495
03/12/2002	564.7918	5.9876	554.7313	578.5405

TABLE 6
Upper bounds for the quality option price (euros) (first study)
(first bid/ask within the minute)

	Average	S.D.	Min.	Max.
10/09/2002	2116.5909	19.8424	2077.8419	2149.6171
17/09/2002	2087.7974	20.8282	2064.9014	2147.7664
24/09/2002	2029.4700	22.4364	1986.8325	2065.1765
01/10/2002	1860.6771	22.0386	1816.3017	1895.0464
08/10/2002	1629.2118	15.9634	1599.3549	1656.0699
15/10/2002	1399.0561	15.7278	1374.0071	1430.9041
22/10/2002	1236.5845	11.2627	1213.8295	1259.9790
29/10/2002	1113.6743	17.5307	1076.6540	1142.7174
05/11/2002	960.9985	14.8429	934.3744	986.9549
12/11/2002	939.7295	13.1983	902.4932	958.8066
19/11/2002	843.3840	7.8334	827.6709	850.0025
26/11/2002	700.4784	14.5163	673.0992	731.4774
03/12/2002	640.0106	15.4790	593.0401	671.1735

TABLE 7
Upper bounds for the quality option price (euros) (first study)
(last bid/ask within the minute)

	Average	S.D.	Min.	Max.
10/09/2002	2116.5513	15.2147	2083.3007	2139.6997
17/09/2002	2088.1819	18,4241	2056.9048	2127.9188
24/09/2002	2027.0708	21.2450	1987.3910	2063.3862
01/10/2002	1857.7477	23.6320	1785.6554	1885.1096
08/10/2002	1628.7483	12.7465	1583.3380	1645.2231
15/10/2002	1400.2210	15.2274	1374.8278	1423.4154
22/10/2002	1236.0131	10.1163	1214.1826	1254.9191
29/10/2002	1110.9136	10.5236	1089.3263	1124.4102
05/11/2002	958.5090	11.5347	933.5231	976.9871
12/11/2002	945.8735	7.4900	936.1172	966.7009
19/11/2002	839.9253	6.1447	829.7519	850.0025
26/11/2002	703.2022	8.1018	693.0733	719.9228
03/12/2002	640.8652	6.1320	630.6490	655.7616

FIGURE 1
Quality option price (euros) (first study)
(September, 2002)
(first bid/ask within the minute)

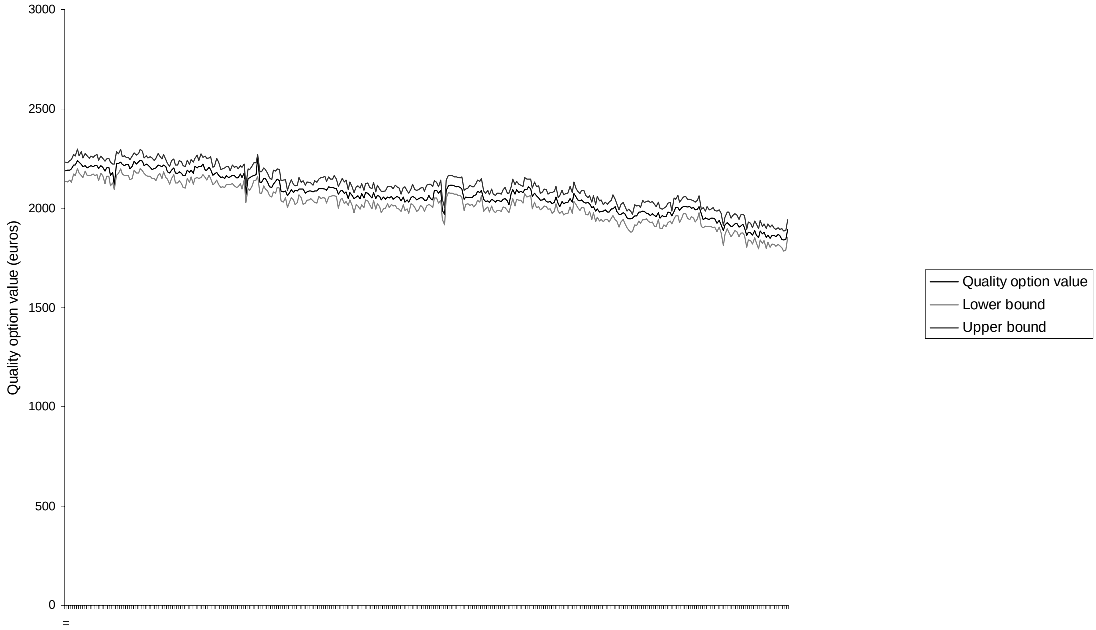

FIGURE 2
Quality option price (euros) (first study)
(October, 2002)
(first bid/ask within the minute)

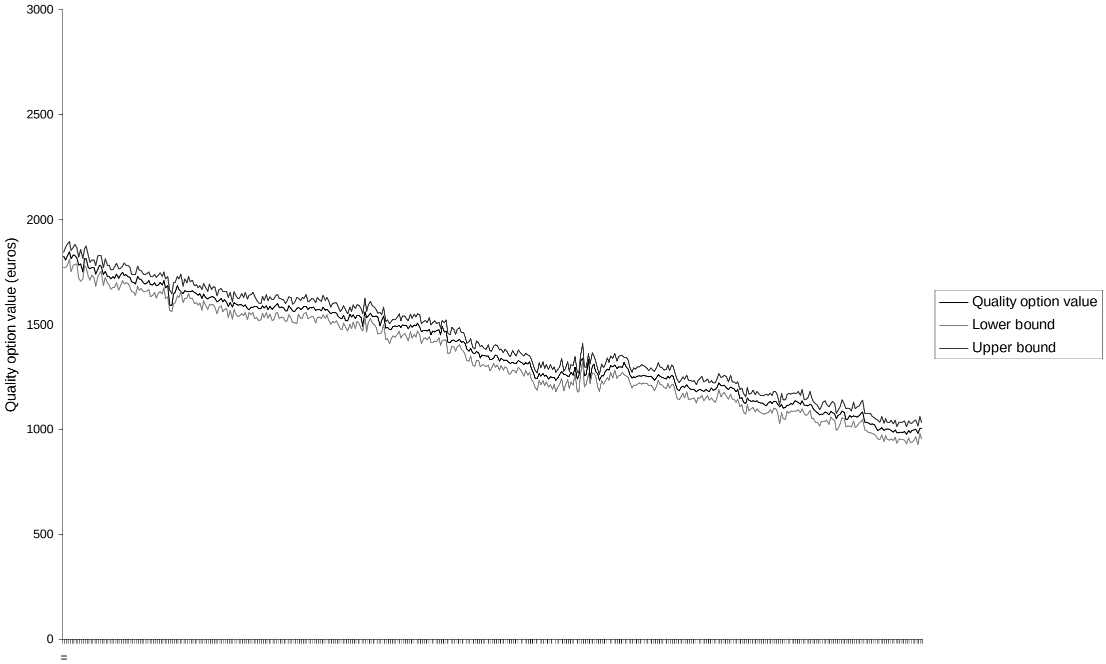

FIGURE 3
Quality option price (euros)
(November, 2002 – December, 6th, 2002) (first study)
(first bid/ask in the minute)

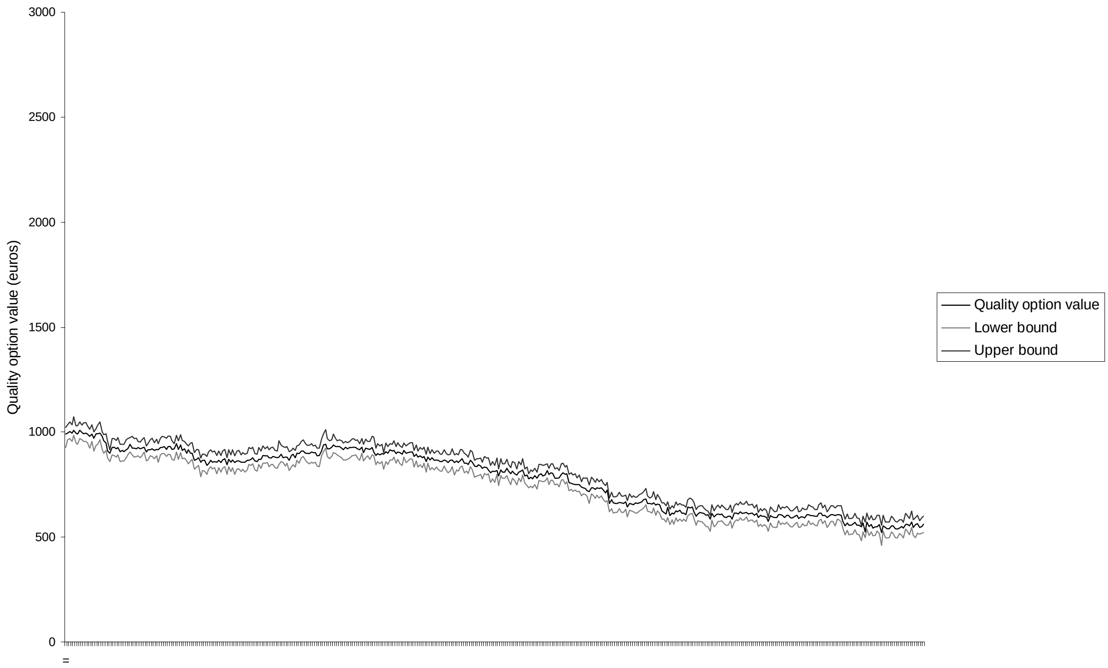

FIGURE 4
Quality option price (euros) (first study)
(September, 2002)
(last bid/ask within the minute)

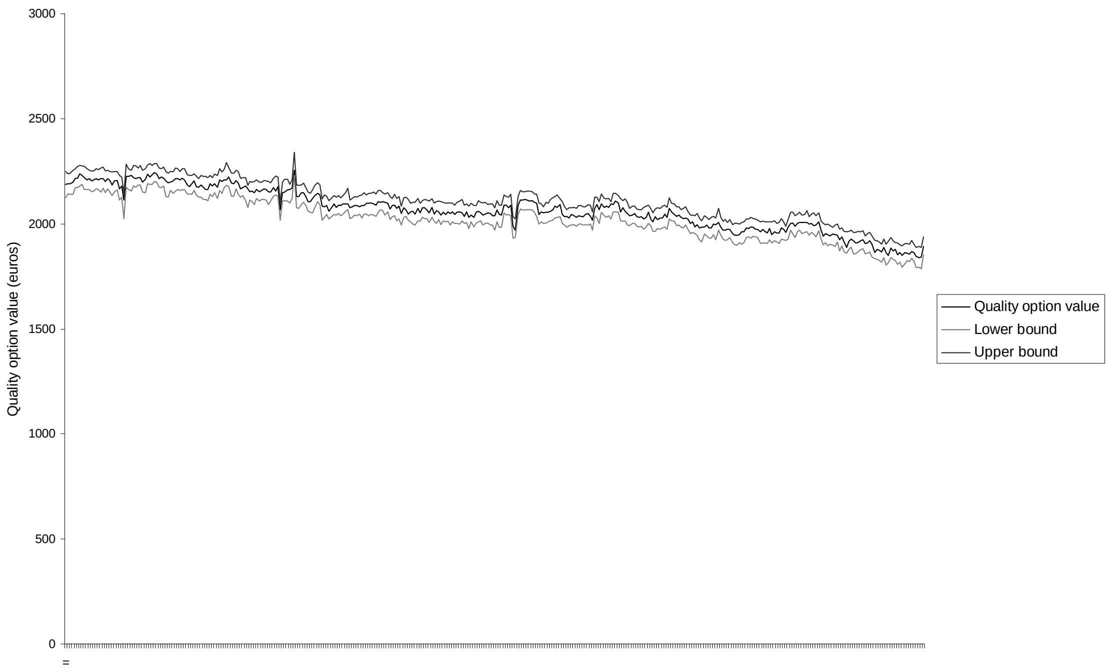

FIGURE 5
Quality option price (euros) (first study)
(October, 2002)
(last bid/ask within the minute)

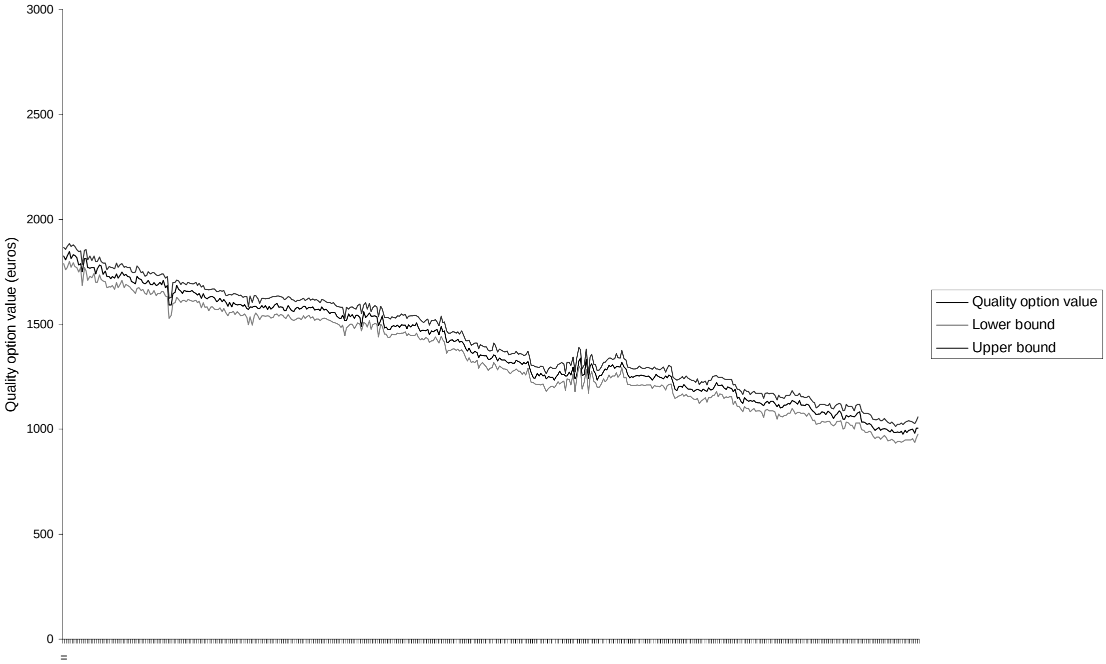

FIGURE 6
Quality option price (euros) (first study)
(November, 2002 – December, 6th, 2002)
(last bid/ask within the minute)

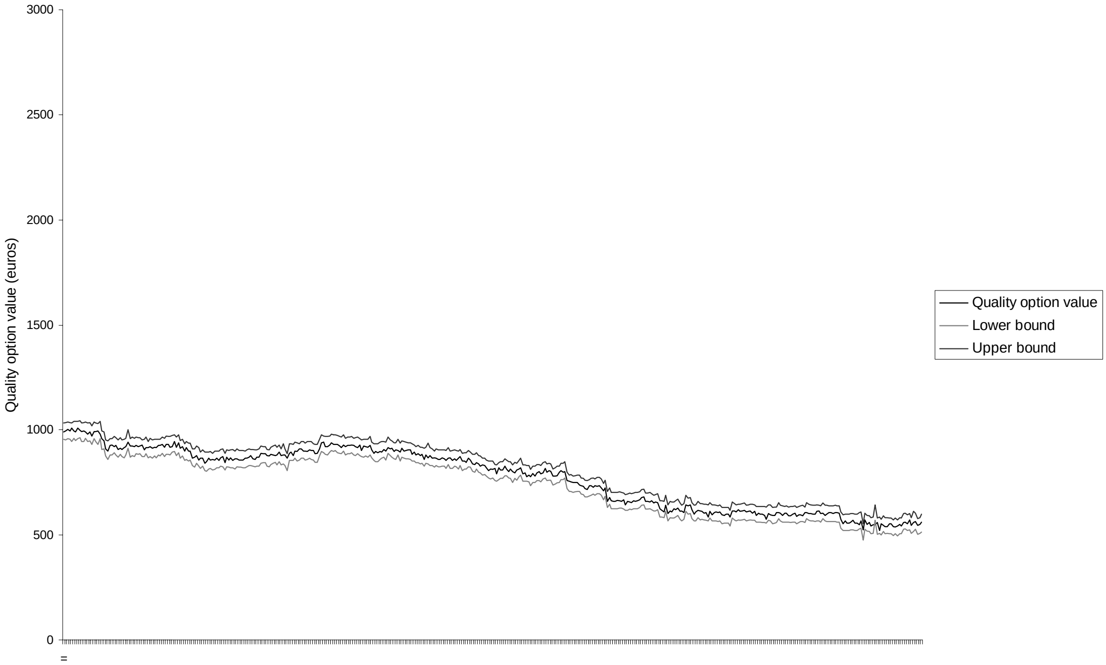

TABLE 8
Deliverable bonds (second study)
(maturity in December, 2005)

Coupon	Coupon payment	Maturity	Conversion factor
4.25%	July, 4th	July, 4^{th}, /2014	0.885160
3.75%	January, 4th	January, 4^{th}, 2015	0.846069
3.25%	July, 4th	July, 4^{th}, 2015	0.803899

TABLE 9
Quality option price (euros) (second study)
(frictionless case)
(strike = 119)

	Average	S.D.	Min.	Max.
02/11/2005	2685.2436	7.6464	2670.9840	2697.2121
04/11/2005	2609.4458	0.0000	2609.4458	2609.4458
07/11/2005	2595.3872	0.0000	2595.3872	2595.3872
08/11/2005	2596.4735	1.7027	2584.7708	2608.1763
09/11/2005	2563.2335	6.2583	2557.2934	2571.8857
10/11/2005	2543.5503	6.7821	2532.3222	2553.8033
11/11/2005	2537.2235	3.7607	2532.0993	2543.8226
14/11/2005	2501.9301	1.7393	2483.6808	2527.6970
15/11/2005	2466.9911	1.7389	2442.4564	2497.2452
16/11/2005	2506.2085	3.5046	2470.6312	2562.5531
18/11/2005	2432.1862	14.3861	2405.2841	2459.0727

TABLE 10
Quality option price (euros) (second study)
(frictionless case, strike = 119.5)

	Average	S.D.	Min.	Max.
02/11/2005	2701.5352	20.6665	2665.9951	2745.2074
03/11/2005	2670.5599	21.0942	2644.2904	2731.8704
04/11/2005	2625.2331	21.1504	2577.2547	2655.8493
07/11/2005	2593.8850	3.2707	2589.4517	2597.9923
08/11/2005	2588.8585	19.6584	2564.1562	2628.6338
09/11/2005	2592.8393	26.9841	2552.3024	2627.7213
10/11/2005	2542.8534	10.2008	2527.3309	2556.1829
11/11/2005	2541.0100	5.4132	2532.0993	2546.4366
14/11/2005	2505.2203	23.7640	2478.6885	2530.0684
15/11/2005	2468.6977	17.8869	2443.6325	2497.2452
16/11/2005	2532.4820	35.6121	2465.6383	2574.5368
17/11/2005	2528.8200	12.4856	2504.3175	2552.4360
18/11/2005	2455.6649	40.4812	2401.6424	2542.1116

TABLE 11
Quality option price (euros) (second study)
(frictionless case, strike = 120)

	Average	S.D.	Min	Max
02/11/2005	2703.0460	25.7214	2665.9951	2761.1857
03/11/2005	2669.5616	19.5506	2642.7196	2731.8704
04/11/2005	2620.1008	25.0137	2572.2652	2655.8493
07/11/2005	2593.9671	5.2874	2584.9337	2600.3044
08/11/2005	2586.5880	21.0829	2559.1655	2623.6430
09/11/2005	2592.3964	24.4400	2555.5303	2616.4628
10/11/2005	2543.7435	0.0000	2543.7435	2543.7435
11/11/2005	2536.9443	3.9127	2532.0993	2541.6816
14/11/2005	2505.2203	21.1881	2483.6808	2527.6970
15/11/2005	2469.5885	17.3130	2446.1309	2494.9599
16/11/2005	2529.3600	36.3622	2424.9241	2574.5368
17/11/2005	2530.6325	14.9181	2509.2234	2552.9103
18/11/2005	2473.6097	42.5412	2424.9241	2539.7598

TABLE 12
Quality option price (euros) (second study)
(frictionless case, strike = 120.5)

	Average	S.D.	Min.	Max.
02/11/2005	2704.4210	23,8382	2675.9730	2761.1857
03/11/2005	2669.1579	22,1661	2644.4256	2731.8704
04/11/2005	2604.5251	0,0000	2604.5251	2604.5251
08/11/2005	2598.1949	0,0000	2598.1949	2598.1949
09/11/2005	2617.3672	13,4810	2603.8892	2637.6272
10/11/2005	2536.0323	7,7112	2528.3212	2543.7435
16/11/2005	2557.1168	11,0188	2540.3592	2574.5368
17/11/2005	2531.0632	14,6291	2509.3107	2552.9103
18/11/2005	2500.5249	33,4205	2434.4586	2537.1181

TABLE 13
Lower bounds (euros) (second study)
(strike = 119)

	Average	S.D.	Min.	Max.
02/11/2005	2613.8875	8.1496	2602.6796	2628.9098
04/11/2005	2553.9559	0.0000	2553.9559	2553.9559
07/11/2005	2530.3113	0.0000	2530.3113	2530.3113
08/11/2005	2533.2961	23.6497	2509.6464	2556.9458
09/11/2005	2496.0572	4.4001	2490.6743	2501.4524
10/11/2005	2488.8216	8.7481	2472.0284	2502.5029
11/11/2005	2478.5991	5.1632	2472.2218	2487.4982
14/11/2005	2446.1114	22.4921	2422.2603	2476.2629
15/11/2005	2410.5681	17.0990	2385.9933	2437.5508
16/11/2005	2444.9190	26.5708	2409.1434	2491.0829
18/11/2005	2369.3658	15.7281	2340.2760	2402.6102

TABLE 14
Lower bonds (euros) (second study)
(strike = 119.5)

	Average	S.D.	Min.	Max.
02/11/2005	2635.2601	17.6532	2610.7920	2671.9310
03/11/2005	2614.1401	21.2338	2586.6702	2671.4502
04/11/2005	2567.1331	21.2655	2516.7745	2595.3747
07/11/2005	2536.6710	5.1827	2529.3650	2542.7507
08/11/2005	2532.6284	20.9490	2505.1769	2579.6590
09/11/2005	2537.1157	27.7953	2496.0448	2571.4685
10/11/2005	2485.1180	11.3727	2462.0460	2499.8916
11/11/2005	2483.7971	6.8919	2472.2218	2490.1119
14/11/2005	2450.4979	24.6179	2422.2603	2476.2629
15/11/2005	2414.3327	19.2756	2386.5132	2447.5359
16/11/2005	2478.4379	36.7870	2409.1434	2524.7400
17/11/2005	2475.5397	14.4666	2447.7912	2495.9112
18/11/2005	2397.1779	41.3105	2350.2630	2485.5522

TABLE 15
Lower bounds (euros) (second study)
(strike = 120)

	Average	S.D.	Min.	Max.
02/11/2005	2647.3592	23.0741	2612.6572	2700.1877
03/11/2005	2613.0656	19.0674	2586.6702	2671.4502
04/11/2005	2566.1848	25.6366	2516.7745	2605.3535
07/11/2005	2536.6570	6.7803	2525.3938	2544.1171
08/11/2005	2532.0941	20.7937	2505.1769	2569.6775
09/11/2005	2534.7991	25.4124	2496.0448	2560.2096
10/11/2005	2487.4522	0.0000	2487.4522	2487.4522
11/11/2005	2465.1398	2.2731	2462.2390	2467.7899
14/11/2005	2448.0017	17.0129	2429.8261	2466.2782
15/11/2005	2398.7092	18.9626	2371.4518	2425.2792
16/11/2005	2473.0546	36.2082	2413.2556	2514.7544
17/11/2005	2476.5458	15.4414	2452.6973	2503.0066
18/11/2005	2416.0036	45.0270	2356.9009	2488.5608

TABLE 16
Lower bounds (euros) (second study)
(Strike = 120.5)

	Average	S.D.	Min.	Max.
02/11/2005	2648.8597	24.6373	2621.8742	2710.1656
03/11/2005	2612.5059	21.9277	2579.4550	2671.4502
04/11/2005	2549.0348	0.0000	2549.0348	2549.0348
08/11/2005	2497.0581	0.0000	2497.0581	2497.0581
09/11/2005	2563.8658	15.7231	2545.7669	2586.3654
10/11/2005	2480.2355	7.2167	2473.0188	2487.4522
16/11/2005	2501.4849	12.5861	2478.5878	2515.6184
17/11/2005	2473.3528	15.8074	2439.0514	2495.9112
18/11/2005	2440.1676	42.4663	2348.2994	2478.9272

TABLE 17
Upper bounds (euros) (second study)
(strike = 119)

	Average	S.D.	Min.	Max.
02/11/2005	2756.6018	8.6659	2739.2905	2767.8455
04/11/2005	2664.9363	0.0000	2664.9363	2664.9363
07/11/2005	2660.4642	0.0000	2660.4642	2660.4642
08/11/2005	2659.6518	0.2448	2659.4070	2659.8965
09/11/2005	2630.4106	16.0480	2618.5425	2653.0979
10/11/2005	2598.2795	5.6524	2587.7509	2605.1039
11/11/2005	2595.8485	3.0913	2591.9776	2600.1472
14/11/2005	2557.7493	15.2036	2545.1017	2579.1314
15/11/2005	2523.4147	17.1664	2498.9198	2556.9405
16/11/2005	2567.4986	37.3877	2526.2455	2634.0237
18/11/2005	2495.0072	18.1703	2470.2933	2545.5806

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TABLE 18
Upper bounds (euros) (second study)
(strike = 119.5)

	Average	S.D.	Min.	Max.
02/11/2005	2767.8118	26.7354	2719.3340	2818.4861
03/11/2005	2726.9805	21.7381	2697.7355	2792.2912
04/11/2005	2683.3339	21.4161	2637.7356	2716.3247
07/11/2005	2651.0995	2.6624	2646.5112	2654.1805
08/11/2005	2645.0890	1.0054	2619.1725	2677.6094
09/11/2005	2648.5633	26.4683	2608.5604	2683.9745
10/11/2005	2600.5893	10.3144	2583.6241	2615.0865
11/11/2005	2598.2234	3.9797	2591.9776	2602.7616
14/11/2005	2559.9431	23.0593	2535.1169	2586.4949
15/11/2005	2523.0631	17.0930	2493.3504	2548.4650
16/11/2005	2586.5265	34.7736	2522.1334	2624.3340
17/11/2005	2582.1007	11.4061	2560.8439	2608.9610
18/11/2005	2514.1523	40.8374	2452.8517	2598.6712

TABLE 19
Upper bounds (euros) (second study)
(strike = 120)

	Average	S.D.	Min.	Max.
02/11/2005	2758.7339	29.1126	2719.3340	2822.1842
03/11/2005	2726.0582	20.5792	2697.7355	2792.2912
04/11/2005	2674.0174	24.6374	2627.7564	2706.3456
07/11/2005	2651.2778	4.1089	2644.4747	2656.4921
08/11/2005	2641.0824	21.8136	2610.3985	2677.6094
09/11/2005	2649.9943	23.7533	2609.6091	2672.7162
10/11/2005	2600.0351	0.0000	2600.0351	2600.0351
11/11/2005	2608.7500	6.7599	2601.9608	2617.9733
14/11/2005	2562.4393	25.4208	2535.1169	2589.1162
15/11/2005	2540.4690	16.0845	2515.9040	2564.6420
16/11/2005	2585.6661	37.0461	2522.1334	2634.3201
17/11/2005	2584.7196	14.9870	2560.8439	2608.9610
18/11/2005	2531.2161	41.1776	2478.4564	2591.3141

TABLE 20
Upper bounds (euros) (second study)
(strike 120.5)

	Average	S.D.	Min.	Max.
02/11/2005	2759.9832	23.3816	2729.3122	2812.2062
03/11/2005	2725.8106	23.2784	2697.7355	2792.2912
04/11/2005	2660.0159	0.0000	2660.0159	2660.0159
08/11/2005	2699.3328	0.0000	2699.3328	2699.3328
09/11/2005	2670.8689	12.2962	2654.6391	2688.8892
10/11/2005	2591.8296	8.2055	2583.6241	2600.0351
16/11/2005	2612.7494	12.3364	2590.1605	2634.3201
17/11/2005	2588.7742	15.7793	2560.8439	2612.8011
18/11/2005	2560.8825	26.1639	2520.6188	2598.6712

FIGURE 7
Quality option price (euros) (second study)
(strike = 119)

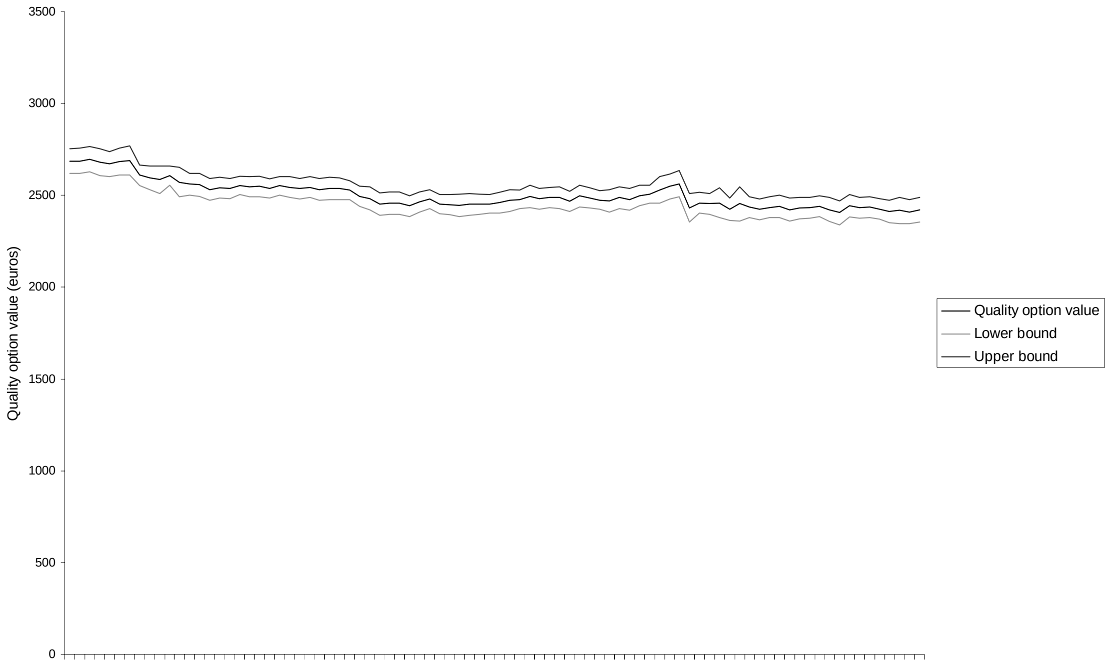


FIGURE 8
Quality option price (euros) (second study)
(strike = 119.5)

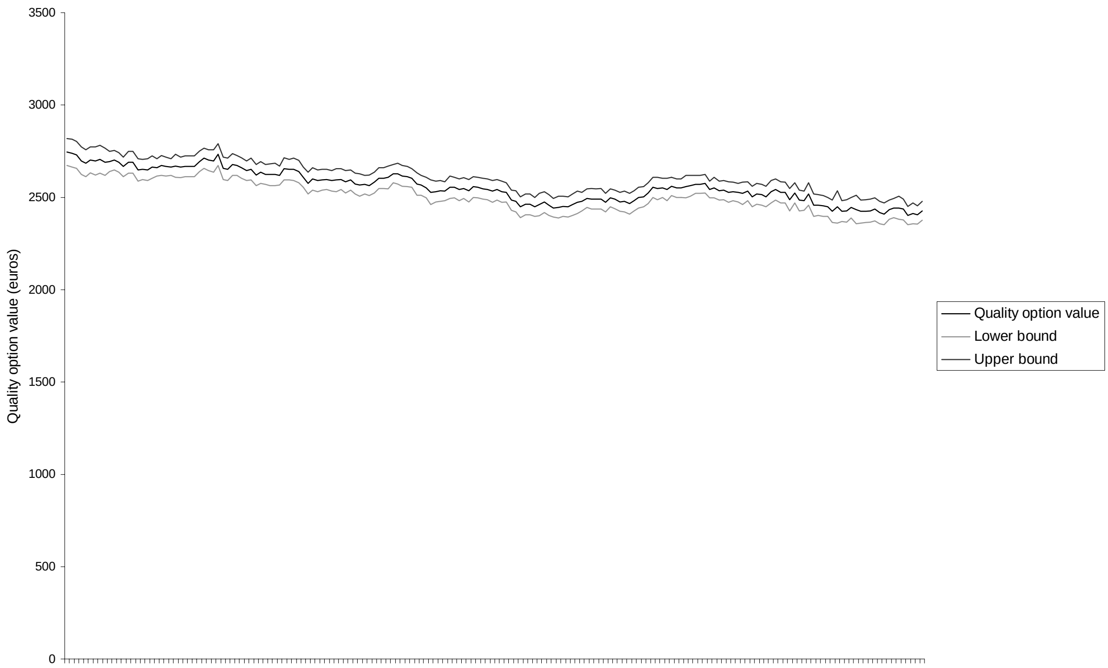


FIGURE 9
Quality option price (euros) (second study)
(strike = 120)

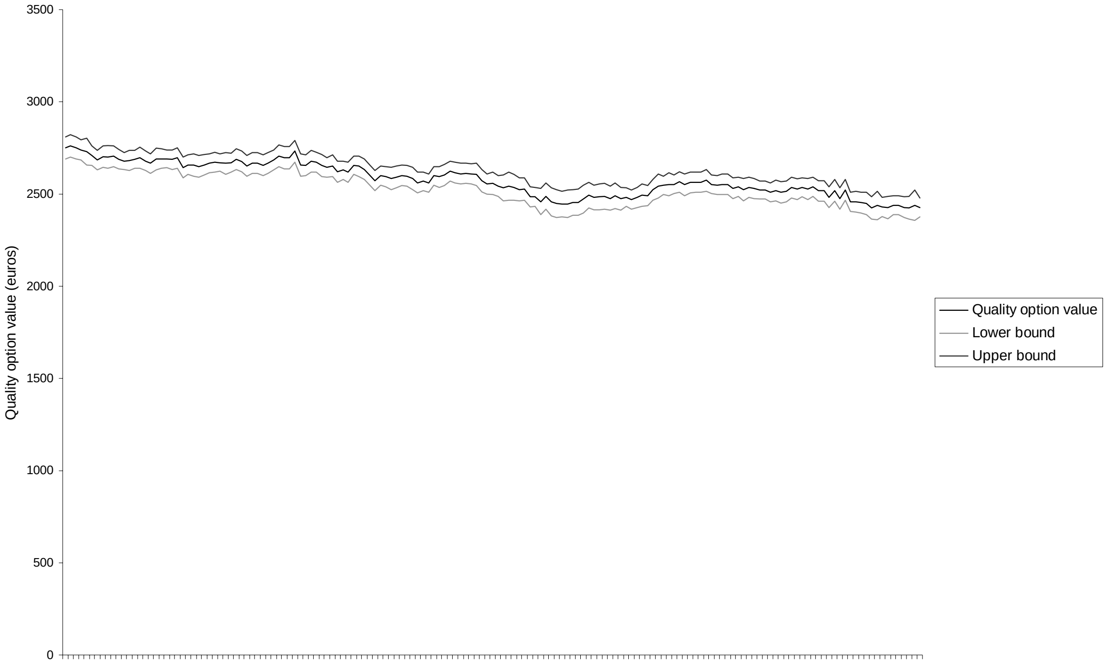
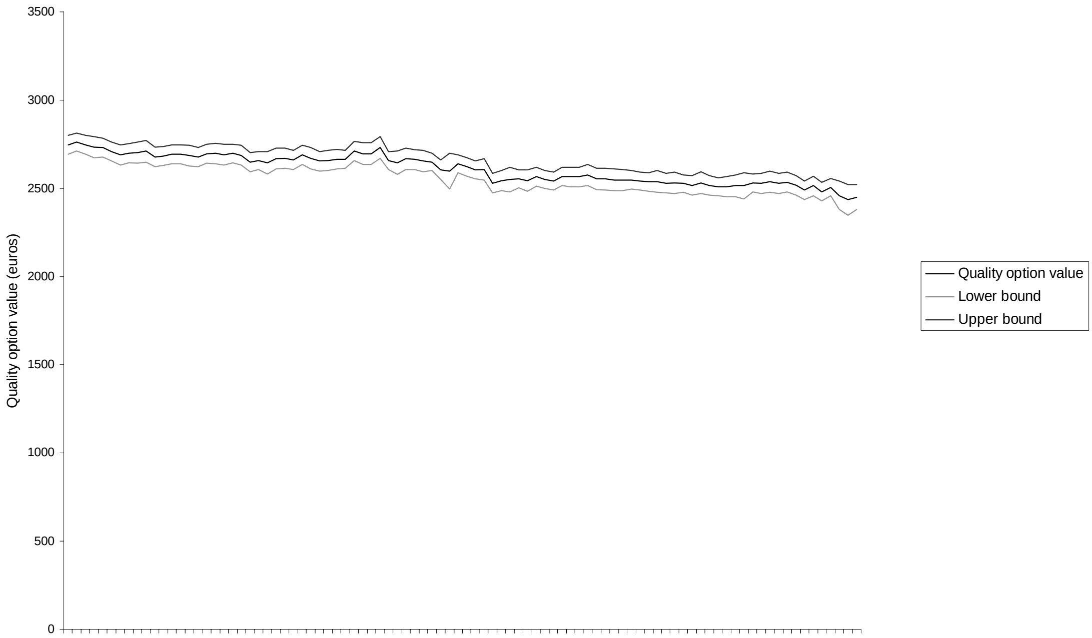


FIGURE 10
Quality option price (euros) (second study)
(strike = 120.5)



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